OCR
Oxford Cambridge and RSA

# Monday 27 June 2016 – Morning

## A2 GCE MATHEMATICS (MEI)

**4764/01** Mechanics 4

**QUESTION PAPER**

*6364314509*

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**Section A** (24 marks)

**1** A car of mass $m$ moves horizontally in a straight line. At time $t$ the car is a distance $x$ from a point O and is moving away from O with speed $v$. There is a force of magnitude $kv^2$, where $k$ is a constant, resisting the motion of the car. The car's engine has a constant power $P$. The terminal speed of the car is $U$.

**(i)** Show that

$$mv^2 \frac{\mathrm{d}v}{\mathrm{d}x} = P\left(1 - \frac{v^3}{U^3}\right).$$

**[3]**

**(ii)** Show that the distance moved while the car accelerates from a speed of $\frac{1}{4}U$ to a speed of $\frac{1}{2}U$ is

$$\frac{mU^3}{3P}\ln A,$$

stating the exact value of the constant $A$. **[6]**

Once the car attains a speed of $\frac{1}{2}U$, no further power is supplied by the car's engine.

**(iii)** Find, in terms of $m$, $P$ and $U$, the time taken for the speed of the car to reduce from $\frac{1}{2}U$ to $\frac{1}{4}U$. **[3]**

**2** A thin rigid rod PQ has length $2a$. Its mass per unit length, $\rho$, is given by $\rho = k\left(1 + \frac{x}{2a}\right)$ where $x$ is the distance from P and $k$ is a positive constant. The mass of the rod is $M$ and the moment of inertia of the rod about an axis through P perpendicular to PQ is $I$.

**(i)** Show that $I = \frac{14}{9}Ma^2$. **[5]**

The rod is initially at rest with Q vertically below P. It is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through P. The rod is struck a horizontal blow perpendicular to the fixed axis at the point where $x = \frac{3}{2}a$. The magnitude of the impulse of this blow is $J$.

**(ii)** Find, in terms of $a$, $J$ and $M$, the initial angular speed of the rod. **[2]**

**(iii)** Find, in terms of $a$, $g$ and $M$, the set of values of $J$ for which the rod makes complete revolutions. **[5]**

**Section B** (48 marks)

**3** Fig. 3 shows a uniform rigid rod AB of length $2a$ and mass $2m$. The rod is freely hinged at A so that it can rotate in a vertical plane. One end of a light inextensible string of length $l$ is attached to B. The string passes over a small smooth fixed pulley at C, where C is vertically above A and AC = $6a$. A particle of mass $\lambda m$, where $\lambda$ is a positive constant, is attached to the other end of the string and hangs freely, vertically below C. The rod makes an angle $\theta$ with the upward vertical, where $0 \leqslant \theta \leqslant \pi$. You may assume that the particle does not interfere with the rod AB or the section of the string BC.

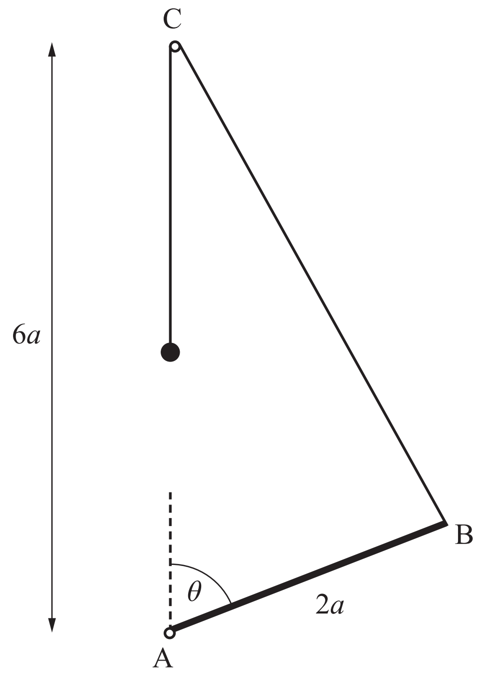


**Fig. 3**

**(i)** Find the potential energy, $V$, of the system relative to a situation in which the rod AB is horizontal, and hence show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right).$$ **[6]**

**(ii)** Show that $\theta = 0$ and $\theta = \pi$ are the only values of $\theta$ for which the system is in equilibrium whatever the value of $\lambda$. **[2]**

**(iii)** Show that, if there is a third value of $\theta$ for which the system is in equilibrium, then $\frac{2}{3} < \lambda < \frac{4}{3}$. **[4]**

**(iv)** Given that there are three positions of equilibrium, establish whether each of these positions is stable or unstable. **[10]**

It is given that, for small values of $\theta$,

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} \approx 2mga\left[\left(\tfrac{3}{2}\lambda - 1\right)\theta - \left(\tfrac{13}{16}\lambda - \tfrac{1}{6}\right)\theta^3\right].$$

.

**(v)** Investigate the stability of the equilibrium position given by $\theta = 0$ in the case when $\lambda = \frac{2}{3}$. **[2]**

**4** A raindrop falls from rest through a stationary cloud. The raindrop has mass $m$ and speed $v$ when it has fallen a distance $x$. You may assume that resistances to motion are negligible.

**(i)** Derive the equation of motion

$$mv\frac{dv}{dx} + v^2\frac{dm}{dx} = mg.$$

**[4]**

Initially the mass of the raindrop is $m_0$. Two different models for the mass of the raindrop are suggested.

In the first model $m = m_0 e^{k_1 x}$, where $k_1$ is a positive constant.

**(ii)** Show that

$$v^2 = \frac{g}{k_1}\left(1 - e^{-2k_1 x}\right),$$

and hence state, in terms of $g$ and $k_1$, the terminal velocity of the raindrop according to this first model. **[7]**

In the second model $m = m_0(1 + k_2 x)$, where $k_2$ is a positive constant.

**(iii)** By considering the expression obtained from differentiating $v^2(1 + k_2 x)^2$ with respect to $x$, show that, for the second model, the equation of motion in part **(i)** may be written as

$$\frac{d}{dx}\left[v^2(1 + k_2 x)^2\right] = 2g(1 + k_2 x)^2.$$

Hence find an expression for $v^2$ in terms of $g$, $k_2$ and $x$ . **[9]**

**(iv)** Suppose that the models give the same value for the speed of the raindrop at the instant when it has doubled its initial mass. Find the exact value of $\frac{k_1}{k_2}$, giving your answer in the form $\frac{a}{b}$ where $a$ and $b$ are integers. **[4]**

**END OF QUESTION PAPER**

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# Monday 27 June 2016 – Morning

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**4764/01** Mechanics 4

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Candidates answer on this Printed Answer Book.

**OCR supplied materials:**
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- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
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**Duration:** 1 hour 30 minutes

| Candidate forename | | Candidate surname | |
|---|---|---|---|

| Centre number | | | | | | Candidate number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

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**Section A** (24 marks)

| | |
|---|---|
| **1 (i)** | |
| **1 (ii)** | |
| | **(answer space continued on next page)** |

| | |
|---|---|
| 1 (ii) | **(continued)** |
| 1 (iii) | |

**2 (i)**

**2 (ii)**

| 2 (iii) | |
|---|---|

**Section B** (48 marks)

| | |
|---|---|
| **3 (i)** | |
| **3 (ii)** | |
| | **(answer space continued on next page)** |

| | |
|---|---|
| 3 (ii) | **(continued)** |
| 3 (iii) | |

| 3 (iv) | |
|---|---|
| | (answer space continued on next page) |

| | |
|---|---|
| 3 (iv) | **(continued)** |

| | |
|---|---|
| **3 (v)** | |

| 4 (i) | |
|---|---|

| 4 (ii) | |
|---|---|

| 4 (iii) | |
|---|---|
| | (answer space continued on next page) |

(answer space continued on next page)

| 4 (iii) | **(continued)** |
|---|---|

**4 (iv)**

## ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

Oxford Cambridge and RSA

# GCE

## Mathematics (MEI)

Unit **4764**: Mechanics 4

Advanced GCE

## Mark Scheme for June 2016

1. **Annotations and abbreviations**

| **Annotation in scoris** | **Meaning** |
|---|---|
| ✓and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| | |
| | |

## 2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

**M**
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**
Mark for a correct result or statement independent of Method marks.

**E**
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

**When a value is given in the paper**
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

**When a value is not given in the paper**
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given.

Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 1 | (i) | | $\frac{P}{U}=kU^2$ | B1 | | |
| | | | $\frac{P}{v}-kv^2=mv\frac{\mathrm{d}v}{\mathrm{d}x}$ | M1 | For use of N2L with any expression for $a$ (3 terms) | |
| | | | $P\left(1-\frac{v^3}{U^3}\right)=mv^2\frac{\mathrm{d}v}{\mathrm{d}x}$ | E1 | | |
| | | | | **[3]** | | |
| 1 | (ii) | | $P\int \mathrm{d}x=mU^3\int\frac{v^2\mathrm{d}v}{U^3-v^3}$ | M1 | Separate variables – condone minor slips | |
| | | | | M1 | Integrate – of the form $Px=k\ln\lvert U^3-v^3\rvert(+c)$ cst. $k$ | |
| | | | $Px=-\frac{1}{3}mU^3\ln\lvert U^3-v^3\rvert(+c)$ | A1 | | |
| | | | $x=0, v=\frac{1}{4}U$ to find $c$ **and** substitute $v=\frac{1}{2}U$ | M1 | Use initial condition – or for use of limits on definite integral | |
| | | | $Px=-\frac{mU^3}{3}\left[\ln\left(U^3-\frac{1}{8}U^3\right)-\ln\left(U^3-\frac{1}{64}U^3\right)\right]$ | A1 | AEF | |
| | | | $x=\frac{mU^3}{3P}\ln\left(\frac{9}{8}\right)$ so $A=\frac{9}{8}$ | E1 | | |
| | | | | **[6]** | | |
| 1 | (iii) | | $-\frac{P}{U^3}v^2=m\frac{\mathrm{d}v}{\mathrm{d}t}\Rightarrow\int\frac{\mathrm{d}v}{v^2}=-\frac{P}{mU^3}\int\mathrm{d}t$ | B1 | Use of $a=\frac{\mathrm{d}v}{\mathrm{d}t}$, power = 0 and separate variables | |
| | | | $-\frac{1}{v}=-\frac{P}{mU^3}t+c$, $t=0, v=\frac{1}{2}U\Rightarrow c=-\frac{2}{U}$ | M1 | Attempt to integrate and use conditions to find $+c$ | |
| | | | $t=\frac{2mU^2}{P}$ | A1 | If left in terms of $k$ eg $t=\frac{2m}{kU}$ then B1M1A0 | |
| | | | | **[3]** | | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 2 | (i) | | $M = k\int_0^{2a}\left(1+\frac{x}{2a}\right)\mathrm{d}x = k\left[x+\frac{x^2}{4a}\right]_0^{2a}$ | M1 | Attempt to integrate (limits not required) |
| | | | $M = 3ka$ | A1 | |
| | | | $I = k\int_0^{2a}\left(1+\frac{x}{2a}\right)x^2\mathrm{d}x$ | B1 | Limits not required |
| | | | $= k\left[\frac{x^3}{3}+\frac{x^4}{8a}\right]_0^{2a}$ | M1 | Attempt to integrate and substitute limits |
| | | | $I = \frac{14}{9}Ma^2$ | E1 | |
| | | | | **[5]** | |
| 2 | (ii) | | $\frac{3}{2}aJ = \left(\frac{14}{9}Ma^2\right)\omega$ | M1 | Conservation of angular momentum |
| | | | $\omega = \frac{27J}{28Ma}$ | A1 | |
| | | | | **[2]** | |
| 2 | (iii) | | $3ka\bar{x} = k\int_0^{2a}\left(x+\frac{x^2}{2a}\right)\mathrm{d}x$ | B1 | Limits not required |
| | | | $= \left[\frac{x^2}{2}+\frac{x^3}{6a}\right]_0^{2a}$ | M1 | Attempt to integrate (limits not required) to find c of m |
| | | | $\bar{x} = \frac{10}{9}a$ | A1 | |
| | | | $-\frac{20}{9}Mga+\frac{1}{2}\left(\frac{14}{9}Ma^2\right)\left(\frac{27J}{28Ma}\right)^2 > 0$ | M1 | Conservation of energy using their $\bar{x}$ (accept $\geq$ or =) must substitute for their $\omega$ - condone errors in PE term |
| | | | $J > \frac{8}{27}M\sqrt{35ga}$ | A1 | AEF – accept $\geq$ |
| | | | | **[5]** | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 3 | (i) | | $BC^2 = (6a)^2 + (2a)^2 - 2(6a)(2a)\cos\theta$ | B1 | $BC = 2a\sqrt{10-6\cos\theta}$ |
| | | | $V = \lambda mg\left(BC - \sqrt{40}a\right) + \ldots$ | M1 | GPE for $\lambda mg$ particle – accept $\lambda mg(6a-(l-BC))$ - where BC is a function of $\theta$ |
| | | | $\ldots + 2mga\cos\theta$ | B1 | GPE for rod |
| | | | | M1 | Differentiate |
| | | | $\frac{dV}{d\theta} = \lambda mga(10-6\cos\theta)^{-\frac{1}{2}}(6\sin\theta) + 2mga(-\sin\theta)$ | A1 | AEF |
| | | | $= 2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right)$ | E1 | www |
| | | | | **[6]** | |
| 3 | (ii) | | $2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right) = 0 \Rightarrow \sin\theta = 0$ or $\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1 = 0$ | M1 | |
| | | | $\sin\theta = 0 \Rightarrow \theta = 0$ or $\pi$ while any other positions are dependent on $\lambda$ | A1 | Must include consideration of $\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1 = 0$ |
| | | | | **[2]** | |
| 3 | (iii) | | $2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right) = 0$ | | Award first two marks for this part if seen in (ii) – regardless of what appears in (iii) |
| | | | so $\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1 = 0$ | M1 | |
| | | | $\Rightarrow \cos\theta = \frac{10-9\lambda^2}{6}$ | A1 | |
| | | | Since $0 < \theta < \pi$ , $-1 < \frac{10-9\lambda^2}{6} < 1$ | M1 | Must state $0 < \theta < \pi$ or $-1 < \cos\theta < 1$ (oe e.g. $4 < 10-6\cos\theta < 16$) – condone $\le$ for the M mark only |
| | | | $\Rightarrow \frac{2}{3} < \lambda < \frac{4}{3}$ | E1 | |
| | | | | **[4]** | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 3 | (iv) | | | M1* | Differentiate using product/quotient rule |
| | | | $\frac{d^2V}{d\theta^2} = 2mga\cos\theta\left(3\lambda(10-6\cos\theta)^{-\frac{1}{2}}-1\right)$ | A1 | |
| | | | $+2mga\sin\theta\left(-\frac{3}{2}\lambda(10-6\cos\theta)^{-\frac{3}{2}}(6\sin\theta)\right)$ | A1 | |
| | | | | M1 dep* | Substituting $\theta=\pi$ into their $V''$ and attempt to simplify |
| | | | $\frac{d^2V}{d\theta^2} = -2mga\left(\frac{3\lambda}{4}-1\right) > 0 \Rightarrow$ stable | A1 | Correct $V''$ and conclusion |
| | | | | M1 dep* | Setting $\frac{3\lambda}{\sqrt{10-6\cos\theta}}-1$ equal to zero in their $V''$ - dependent on first M mark |
| | | | $\frac{d^2V}{d\theta^2} = -18mga\lambda\sin^2\theta(10-6\cos\theta)^{-3/2} < 0 \Rightarrow$ unstable | A1 | Correct $V''$ and conclusion |
| | | | | M1 dep* | Substituting $\theta=0$ into their $V''$ and attempt to simplify – dependent on first M mark |
| | | | $\frac{d^2V}{d\theta^2} = 2mga\left(\frac{3\lambda}{2}-1\right) > 0 \Rightarrow$ stable | A1 | Correct $V''$ and conclusion |
| | | | | A1 | Clear justification of each value of $V''$ for the three cases with particular reference made to the values of $\lambda$ in the interval $\frac{2}{3}<\lambda<\frac{4}{3}$ (dependent on all previous marks earned in this part) |
| | | | | **[10]** | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 3 | (v) | | $\frac{\mathrm{d}V}{\mathrm{d}\theta} = -\frac{3}{4}mga\theta^3 + \cdots$ | M1 | Substituting $\lambda = \frac{2}{3}$ and using first derivative test **only** | |
| | | | When $\theta = 0^-, \frac{\mathrm{d}V}{\mathrm{d}\theta} > 0$ **and** when $\theta = 0^+, \frac{\mathrm{d}V}{\mathrm{d}\theta} < 0$ $\therefore$ unstable at $\lambda = \frac{2}{3}$ | A1 | CAO – allow accurate sketch of the form $y = -x^3$ | |
| | | | | **[2]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 4 | (i) | | | M1 | Impulse = change in momentum | |
| | | | $(m+\delta m)(v+\delta v)-(mv+\delta m(0))=(m+\delta m)g\delta t$ | A1 | Condone lack of $\delta m$ on rhs | |
| | | | $m\frac{\delta v}{\delta t}+v\frac{\delta m}{\delta t}+\frac{\delta m}{\delta t}\frac{\delta v}{\delta t}\delta t=mg+g\frac{\delta m}{\delta t}\delta t$ | M1 | Form differential equation | |
| | | | $mv\frac{\mathrm{d}v}{\mathrm{d}x}+v^2\frac{\mathrm{d}m}{\mathrm{d}x}=mg$ | E1 | Complete argument | |
| | | | | **[4]** | | |
| | | **OR** | $\frac{\mathrm{d}}{\mathrm{d}t}(mv)=mg$ | M1A1 | | |
| | | | $\frac{\mathrm{d}}{\mathrm{d}t}(mv)=mg\Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t}+v\frac{\mathrm{d}m}{\mathrm{d}t}=mg$ | A1 | | |
| | | | $mv\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}m}{\mathrm{d}t}\ \ mv\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}m}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}=mg\Rightarrow mv\frac{\mathrm{d}v}{\mathrm{d}x}+v^2\frac{\mathrm{d}m}{\mathrm{d}x}=mg$ | E1 | | |
| 4 | (ii) | | $\frac{\mathrm{d}m}{\mathrm{d}x}=m_0k_1\mathrm{e}^{k_1x}$ | B1 | | |
| | | | $v\frac{\mathrm{d}v}{\mathrm{d}x}=g-k_1v^2\Rightarrow\int\frac{v\mathrm{d}v}{g-k_1v^2}=\int\mathrm{d}x$ | M1 | Substitute $m$ and $\frac{\mathrm{d}m}{\mathrm{d}t}$ and separate variables | |
| | | | | M1 | Integrate of the form $\lambda\ln\left(g-k_1v^2\right)=x(+c)$ cst. $\lambda$ | |
| | | | $-\frac{1}{2k_1}\ln\left(g-k_1v^2\right)=x+c$ | A1 | Including $+c$ | |
| | | | $x=0, v=0\Rightarrow c=-\frac{1}{2k_1}\ln g$ | M1 | Use conditions to obtain $c$ and attempt to make $v^2$ the subject | |
| | | | $v^2=\frac{g}{k_1}\left(1-\mathrm{e}^{-2k_1x}\right)$ | E1 | www after correct integration | |
| | | | Terminal velocity is $\sqrt{\frac{g}{k_1}}$ | B1 | | |
| | | | | **[7]** | | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 4 | (iii) | | $\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1+k_2x)^2\right]=$ | M1 | Differentiate $v^2(1+k_2x)^2$ using product rule |
| | | | $2v\frac{\mathrm{d}v}{\mathrm{d}x}(1+k_2x)^2+2k_2v^2(1+k_2x)$ | A1 A1 | A1 for each term |
| | | | $mv\frac{\mathrm{d}v}{\mathrm{d}x}+v^2\frac{\mathrm{d}m}{\mathrm{d}x}=mg$ and $m=m_0(1+k_2x)$ | | |
| | | | $\frac{\mathrm{d}m}{\mathrm{d}x}=k_2m_0 \Rightarrow (1+k_2x)v\frac{\mathrm{d}v}{\mathrm{d}x}+k_2v^2=(1+k_2x)g$ | B1 | Correct $\frac{\mathrm{d}m}{\mathrm{d}x}$ and substitute both $m$ and $\frac{\mathrm{d}m}{\mathrm{d}x}$ into DE |
| | | | $\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1+k_2x)^2\right]=2(1+k_2x)\left((1+k_2x)g-k_2v^2\right)+2k_2v^2(1+k_2x)$ | | |
| | | | $\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1+k_2x)^2\right]=2g(1+k_2x)^2$ | E1 | Working must be clear (dependent on all previous marks) |
| | | | $v^2(1+k_2x)^2=2g\int(1+k_2x)^2\,\mathrm{d}x$ | B1 | |
| | | | $=2g\left(\frac{(1+k_2x)^3}{3k_2}\right)(+c)$ | B1 | |
| | | | $v=0, x=0 \Rightarrow c=-\frac{2g}{3k_2}$ | M1 | Use conditions to find $+c$ or limits from 0 to $x$ on definite integral |
| | | | $v^2=\frac{2g}{3k_2}\left[1+k_2x-\frac{1}{(1+k_2x)^2}\right]$ | A1 | oe eg $v^2=\frac{2gx\left(1+k_2x+\frac{1}{3}k_2{}^2x^2\right)}{(1+k_2x)^2}$ |
| | | | | **[9]** | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 4 | (iv) | | First model: $x = \frac{1}{k_1}\ln 2$ | B1 | |
| | | | Second model: $x = \frac{1}{k_2}$ | B1 | |
| | | | $\frac{2g}{3k_2}\left(1+k_2\left(\frac{1}{k_2}\right)\right) - \frac{2g}{3k_2\left(1+k_2\left(\frac{1}{k_2}\right)\right)^2} = \frac{g}{k_1}\left(1-\mathrm{e}^{-2k_1\left(\frac{1}{k_1}\ln 2\right)}\right)$ | M1 | Substitute both $x$ values and equate |
| | | | $\frac{k_1}{k_2} = \frac{9}{14}$ | A1 | |
| | | | | **[4]** | |
| | | **OR** | First model: $\mathrm{e}^{k_1 x} = 2$ | B1 | |
| | | | Second model: $k_2 x = 1$ | B1 | |
| | | | $\frac{3g}{4k_1} = \frac{7g}{6k_2}$ | M1 | Substituting for both and equating expressions for $v^2$ for reference $v^2 = \frac{3g}{4k_1}$ (1st model) and $v^2 = \frac{7g}{6k_2}$ (2nd model) |
| | | | $\frac{k_1}{k_2} = \frac{9}{14}$ | A1 | |

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PART OF THE CAMBRIDGE ASSESSMENT GROUP

GCE

# Mathematics (MEI)

Unit **4764**: Mechanics 4

Advanced GCE

## Mark Scheme for June 2016

1. **Annotations and abbreviations**

| **Annotation in scoris** | **Meaning** |
|---|---|
| ✓and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| | |
| | |

## 2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

**M**
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**
Mark for a correct result or statement independent of Method marks.

**E**
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

**When a value is given in the paper**
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

**When a value is not given in the paper**
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given.

Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 1 | (i) | | $\frac{P}{U}=kU^2$ | B1 | |
| | | | $\frac{P}{v}-kv^2=mv\frac{\mathrm{d}v}{\mathrm{d}x}$ | M1 | For use of N2L with any expression for $a$ (3 terms) |
| | | | $P\left(1-\frac{v^3}{U^3}\right)=mv^2\frac{\mathrm{d}v}{\mathrm{d}x}$ | E1 | |
| | | | | **[3]** | |
| 1 | (ii) | | $P\int \mathrm{d}x=mU^3\int\frac{v^2\mathrm{d}v}{U^3-v^3}$ | M1 | Separate variables – condone minor slips |
| | | | | M1 | Integrate – of the form $Px=k\ln\left\|U^3-v^3\right\|(+c)$ cst. $k$ |
| | | | $Px=-\frac{1}{3}mU^3\ln\left\|U^3-v^3\right\|(+c)$ | A1 | |
| | | | $x=0, v=\frac{1}{4}U$ to find $c$ **and** substitute $v=\frac{1}{2}U$ | M1 | Use initial condition – or for use of limits on definite integral |
| | | | $Px=-\frac{mU^3}{3}\left[\ln\left(U^3-\frac{1}{8}U^3\right)-\ln\left(U^3-\frac{1}{64}U^3\right)\right]$ | A1 | AEF |
| | | | $x=\frac{mU^3}{3P}\ln\left(\frac{9}{8}\right)$ so $A=\frac{9}{8}$ | E1 | |
| | | | | **[6]** | |
| 1 | (iii) | | $-\frac{P}{U^3}v^2=m\frac{\mathrm{d}v}{\mathrm{d}t}\Rightarrow\int\frac{\mathrm{d}v}{v^2}=-\frac{P}{mU^3}\int\mathrm{d}t$ | B1 | Use of $a=\frac{\mathrm{d}v}{\mathrm{d}t}$, power = 0 and separate variables |
| | | | $-\frac{1}{v}=-\frac{P}{mU^3}t+c$ , $t=0, v=\frac{1}{2}U\Rightarrow c=-\frac{2}{U}$ | M1 | Attempt to integrate and use conditions to find $+c$ |
| | | | $t=\frac{2mU^2}{P}$ | A1 | If left in terms of $k$ eg $t=\frac{2m}{kU}$ then B1M1A0 |
| | | | | **[3]** | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 2 | (i) | | $M = k\int_0^{2a}\left(1+\frac{x}{2a}\right)\mathrm{d}x = k\left[x+\frac{x^2}{4a}\right]_0^{2a}$ | M1 | Attempt to integrate (limits not required) | |
| | | | $M = 3ka$ | A1 | | |
| | | | $I = k\int_0^{2a}\left(1+\frac{x}{2a}\right)x^2\mathrm{d}x$ | B1 | Limits not required | |
| | | | $= k\left[\frac{x^3}{3}+\frac{x^4}{8a}\right]_0^{2a}$ | M1 | Attempt to integrate and substitute limits | |
| | | | $I = \frac{14}{9}Ma^2$ | E1 | | |
| | | | | **[5]** | | |
| 2 | (ii) | | $\frac{3}{2}aJ = \left(\frac{14}{9}Ma^2\right)\omega$ | M1 | Conservation of angular momentum | |
| | | | $\omega = \frac{27J}{28Ma}$ | A1 | | |
| | | | | **[2]** | | |
| 2 | (iii) | | $3ka\bar{x} = k\int_0^{2a}\left(x+\frac{x^2}{2a}\right)\mathrm{d}x$ | B1 | Limits not required | |
| | | | $= \left[\frac{x^2}{2}+\frac{x^3}{6a}\right]_0^{2a}$ | M1 | Attempt to integrate (limits not required) to find c of m | |
| | | | $\bar{x} = \frac{10}{9}a$ | A1 | | |
| | | | $-\frac{20}{9}Mga+\frac{1}{2}\left(\frac{14}{9}Ma^2\right)\left(\frac{27J}{28Ma}\right)^2 > 0$ | M1 | Conservation of energy using their $\bar{x}$ (accept $\geq$ or =) must substitute for their $\omega$ - condone errors in PE term | |
| | | | $J > \frac{8}{27}M\sqrt{35ga}$ | A1 | AEF – accept $\geq$ | |
| | | | | **[5]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 3 | (i) | | $BC^2 = (6a)^2 + (2a)^2 - 2(6a)(2a)\cos\theta$ | B1 | $BC = 2a\sqrt{10-6\cos\theta}$ | |
| | | | $V = \lambda mg\left(BC - \sqrt{40}a\right) + \ldots$ | M1 | GPE for $\lambda mg$ particle – accept $\lambda mg(6a-(l-BC))$ - where BC is a function of $\theta$ | |
| | | | $\ldots + 2mga\cos\theta$ | B1 | GPE for rod | |
| | | | | M1 | Differentiate | |
| | | | $\frac{dV}{d\theta} = \lambda mga\left(10-6\cos\theta\right)^{-\frac{1}{2}}\left(6\sin\theta\right) + 2mga\left(-\sin\theta\right)$ | A1 | AEF | |
| | | | $= 2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right)$ | E1 | www | |
| | | | | **[6]** | | |
| 3 | (ii) | | $2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right) = 0 \Rightarrow \sin\theta = 0$ or $\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1 = 0$ | M1 | | |
| | | | $\sin\theta = 0 \Rightarrow \theta = 0$ or $\pi$ while any other positions are dependent on $\lambda$ | A1 | Must include consideration of $\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1 = 0$ | |
| | | | | **[2]** | | |
| 3 | (iii) | | $2mga\sin\theta\left(\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1\right) = 0$ | | Award first two marks for this part if seen in (ii) – regardless of what appears in (iii) | |
| | | | so $\frac{3\lambda}{\sqrt{10-6\cos\theta}} - 1 = 0$ | M1 | | |
| | | | $\Rightarrow \cos\theta = \frac{10-9\lambda^2}{6}$ | A1 | | |
| | | | Since $0 < \theta < \pi$ , $-1 < \frac{10-9\lambda^2}{6} < 1$ | M1 | Must state $0 < \theta < \pi$ or $-1 < \cos\theta < 1$ (oe e.g. $4 < 10 - 6\cos\theta < 16$) – condone $\le$ for the M mark only | |
| | | | $\Rightarrow \frac{2}{3} < \lambda < \frac{4}{3}$ | E1 | | |
| | | | | **[4]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 3 | (iv) | | $\frac{d^2V}{d\theta^2} = 2mga\cos\theta\left(3\lambda(10-6\cos\theta)^{-\frac{1}{2}}-1\right)$ <br> $+2mga\sin\theta\left(-\frac{3}{2}\lambda(10-6\cos\theta)^{-\frac{3}{2}}(6\sin\theta)\right)$ | M1* <br> A1 <br> A1 | Differentiate using product/quotient rule | |
| | | | | M1 dep* | Substituting $\theta=\pi$ into their $V''$ and attempt to simplify | |
| | | | $\frac{d^2V}{d\theta^2} = -2mga\left(\frac{3\lambda}{4}-1\right) > 0 \Rightarrow$ stable | A1 | Correct $V''$ and conclusion | |
| | | | | M1 dep* | Setting $\frac{3\lambda}{\sqrt{10-6\cos\theta}}-1$ equal to zero in their $V''$ - dependent on first M mark | |
| | | | $\frac{d^2V}{d\theta^2} = -18mga\lambda\sin^2\theta(10-6\cos\theta)^{-3/2} < 0 \Rightarrow$ unstable | A1 | Correct $V''$ and conclusion | |
| | | | | M1 dep* | Substituting $\theta=0$ into their $V''$ and attempt to simplify – dependent on first M mark | |
| | | | $\frac{d^2V}{d\theta^2} = 2mga\left(\frac{3\lambda}{2}-1\right) > 0 \Rightarrow$ stable | A1 | Correct $V''$ and conclusion | |
| | | | | A1 | Clear justification of each value of $V''$ for the three cases with particular reference made to the values of $\lambda$ in the interval $\frac{2}{3} < \lambda < \frac{4}{3}$ (dependent on all previous marks earned in this part) | |
| | | | | **[10]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 3 | (v) | | $\frac{\mathrm{d}V}{\mathrm{d}\theta} = -\frac{3}{4}mga\theta^3 + \cdots$ | M1 | Substituting $\lambda = \frac{2}{3}$ and using first derivative test **only** | |
| | | | When $\theta = 0^-, \frac{\mathrm{d}V}{\mathrm{d}\theta} > 0$ **and** when $\theta = 0^+, \frac{\mathrm{d}V}{\mathrm{d}\theta} < 0$ $\therefore$ unstable at $\lambda = \frac{2}{3}$ | A1 | CAO – allow accurate sketch of the form $y = -x^3$ | |
| | | | | **[2]** | | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 4 | (i) | | | M1 | Impulse = change in momentum |
| | | | $(m+\delta m)(v+\delta v)-(mv+\delta m(0))=(m+\delta m)g\delta t$ | A1 | Condone lack of $\delta m$ on rhs |
| | | | $m\frac{\delta v}{\delta t}+v\frac{\delta m}{\delta t}+\frac{\delta m}{\delta t}\frac{\delta v}{\delta t}\delta t=mg+g\frac{\delta m}{\delta t}\delta t$ | M1 | Form differential equation |
| | | | $mv\frac{\mathrm{d}v}{\mathrm{d}x}+v^2\frac{\mathrm{d}m}{\mathrm{d}x}=mg$ | E1 | Complete argument |
| | | | | **[4]** | |
| | | OR | $\frac{\mathrm{d}}{\mathrm{d}t}(mv)=mg$ | M1A1 | |
| | | | $\frac{\mathrm{d}}{\mathrm{d}t}(mv)=mg\Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t}+v\frac{\mathrm{d}m}{\mathrm{d}t}=mg$ | A1 | |
| | | | $mv\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}m}{\mathrm{d}t}\ \ mv\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}m}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}=mg\Rightarrow mv\frac{\mathrm{d}v}{\mathrm{d}x}+v^2\frac{\mathrm{d}m}{\mathrm{d}x}=mg$ | E1 | |
| 4 | (ii) | | $\frac{\mathrm{d}m}{\mathrm{d}x}=m_0k_1\mathrm{e}^{k_1x}$ | B1 | |
| | | | $v\frac{\mathrm{d}v}{\mathrm{d}x}=g-k_1v^2\Rightarrow\int\frac{v\mathrm{d}v}{g-k_1v^2}=\int\mathrm{d}x$ | M1 | Substitute *m* and $\frac{\mathrm{d}m}{\mathrm{d}t}$ and separate variables |
| | | | | M1 | Integrate of the form $\lambda\ln\left(g-k_1v^2\right)=x(+c)$ cst. $\lambda$ |
| | | | $-\frac{1}{2k_1}\ln\left(g-k_1v^2\right)=x+c$ | A1 | Including $+c$ |
| | | | $x=0,v=0\Rightarrow c=-\frac{1}{2k_1}\ln g$ | M1 | Use conditions to obtain $c$ and attempt to make $v^2$ the subject |
| | | | $v^2=\frac{g}{k_1}\left(1-\mathrm{e}^{-2k_1x}\right)$ | E1 | www after correct integration |
| | | | Terminal velocity is $\sqrt{\frac{g}{k_1}}$ | B1 | |
| | | | | **[7]** | |

| Question | | Answer | Marks | Guidance |
|---|---|---|---|---|
| 4 | (iii) | $\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1+k_2x)^2\right]=$ | M1 | Differentiate $v^2(1+k_2x)^2$ using product rule |
| | | $2v\frac{\mathrm{d}v}{\mathrm{d}x}(1+k_2x)^2+2k_2v^2(1+k_2x)$ | A1 A1 | A1 for each term |
| | | $mv\frac{\mathrm{d}v}{\mathrm{d}x}+v^2\frac{\mathrm{d}m}{\mathrm{d}x}=mg$ and $m=m_0(1+k_2x)$ | | |
| | | $\frac{\mathrm{d}m}{\mathrm{d}x}=k_2m_0 \Rightarrow (1+k_2x)v\frac{\mathrm{d}v}{\mathrm{d}x}+k_2v^2=(1+k_2x)g$ <br> $\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1+k_2x)^2\right]=2(1+k_2x)\left((1+k_2x)g-k_2v^2\right)+2k_2v^2(1+k_2x)$ | B1 | Correct $\frac{\mathrm{d}m}{\mathrm{d}x}$ and substitute both $m$ and $\frac{\mathrm{d}m}{\mathrm{d}x}$ into DE |
| | | $\frac{\mathrm{d}}{\mathrm{d}x}\left[v^2(1+k_2x)^2\right]=2g(1+k_2x)^2$ | E1 | Working must be clear (dependent on all previous marks) |
| | | $v^2(1+k_2x)^2=2g\int(1+k_2x)^2\,\mathrm{d}x$ | B1 | |
| | | $=2g\left(\frac{(1+k_2x)^3}{3k_2}\right)(+c)$ | B1 | |
| | | $v=0, x=0 \Rightarrow c=-\frac{2g}{3k_2}$ | M1 | Use conditions to find $+c$ or limits from 0 to $x$ on definite integral |
| | | $v^2=\frac{2g}{3k_2}\left[1+k_2x-\frac{1}{(1+k_2x)^2}\right]$ | A1 | oe eg $v^2=\frac{2gx\left(1+k_2x+\frac{1}{3}k_2{}^2x^2\right)}{(1+k_2x)^2}$ |
| | | | **[9]** | |

| Question | | | Answer | Marks | Guidance |
|---|---|---|---|---|---|
| 4 | (iv) | | First model: $x = \frac{1}{k_1}\ln 2$ | B1 | |
| | | | Second model: $x = \frac{1}{k_2}$ | B1 | |
| | | | $\frac{2g}{3k_2}\left(1+k_2\left(\frac{1}{k_2}\right)\right) - \frac{2g}{3k_2\left(1+k_2\left(\frac{1}{k_2}\right)\right)^2} = \frac{g}{k_1}\left(1-\mathrm{e}^{-2k_1\left(\frac{1}{k_1}\ln 2\right)}\right)$ | M1 | Substitute both $x$ values and equate |
| | | | $\frac{k_1}{k_2} = \frac{9}{14}$ | A1 | |
| | | | | **[4]** | |
| | | **OR** | First model: $\mathrm{e}^{k_1 x} = 2$ | B1 | |
| | | | Second model: $k_2 x = 1$ | B1 | |
| | | | $\frac{3g}{4k_1} = \frac{7g}{6k_2}$ | M1 | Substituting for both and equating expressions for $v^2$ for reference $v^2 = \frac{3g}{4k_1}$ (1st model) and $v^2 = \frac{7g}{6k_2}$ (2nd model) |
| | | | $\frac{k_1}{k_2} = \frac{9}{14}$ | A1 | |

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**Facsimile: 01223 552553**

GCE Mathematics (MEI)

| | | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|---|
| 4751 | 01 | C1 – MEI Introduction to advanced mathematics (AS) | Raw | 72 | 63 | 57 | 52 | 47 | 42 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4752 | 01 | C2 – MEI Concepts for advanced mathematics (AS) | Raw | 72 | 56 | 49 | 42 | 35 | 29 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4753 | 01 | (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper | Raw | 72 | 58 | 52 | 47 | 42 | 36 | 0 |
| 4753 | 02 | (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework | Raw | 18 | 15 | 13 | 11 | 9 | 8 | 0 |
| 4753 | 82 | (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark | Raw | 18 | 15 | 13 | 11 | 9 | 8 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4754 | 01 | C4 – MEI Applications of advanced mathematics (A2) | Raw | 90 | 64 | 57 | 51 | 45 | 39 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4755 | 01 | FP1 – MEI Further concepts for advanced mathematics (AS) | Raw | 72 | 59 | 53 | 48 | 43 | 38 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4756 | 01 | FP2 – MEI Further methods for advanced mathematics (A2) | Raw | 72 | 60 | 54 | 48 | 43 | 38 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4757 | 01 | FP3 – MEI Further applications of advanced mathematics (A2) | Raw | 72 | 60 | 54 | 49 | 44 | 39 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4758 | 01 | (DE) MEI Differential Equations with Coursework: Written Paper | Raw | 72 | 67 | 61 | 55 | 49 | 43 | 0 |
| 4758 | 02 | (DE) MEI Differential Equations with Coursework: Coursework | Raw | 18 | 15 | 13 | 11 | 9 | 8 | 0 |
| 4758 | 82 | (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark | Raw | 18 | 15 | 13 | 11 | 9 | 8 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4761 | 01 | M1 – MEI Mechanics 1 (AS) | Raw | 72 | 58 | 50 | 43 | 36 | 29 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4762 | 01 | M2 – MEI Mechanics 2 (A2) | Raw | 72 | 59 | 53 | 47 | 41 | 36 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4763 | 01 | M3 – MEI Mechanics 3 (A2) | Raw | 72 | 60 | 53 | 46 | 40 | 34 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4764 | 01 | M4 – MEI Mechanics 4 (A2) | Raw | 72 | 55 | 48 | 41 | 34 | 27 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4766 | 01 | S1 – MEI Statistics 1 (AS) | Raw | 72 | 59 | 52 | 46 | 40 | 34 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4767 | 01 | S2 – MEI Statistics 2 (A2) | Raw | 72 | 60 | 55 | 50 | 45 | 40 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4768 | 01 | S3 – MEI Statistics 3 (A2) | Raw | 72 | 60 | 54 | 48 | 42 | 37 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4769 | 01 | S4 – MEI Statistics 4 (A2) | Raw | 72 | 56 | 49 | 42 | 35 | 28 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4771 | 01 | D1 – MEI Decision mathematics 1 (AS) | Raw | 72 | 48 | 43 | 38 | 34 | 30 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4772 | 01 | D2 – MEI Decision mathematics 2 (A2) | Raw | 72 | 55 | 50 | 45 | 40 | 36 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4773 | 01 | DC – MEI Decision mathematics computation (A2) | Raw | 72 | 46 | 40 | 34 | 29 | 24 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4776 | 01 | (NM) MEI Numerical Methods with Coursework: Written Paper | Raw | 72 | 55 | 49 | 44 | 39 | 33 | 0 |
| 4776 | 02 | (NM) MEI Numerical Methods with Coursework: Coursework | Raw | 18 | 14 | 12 | 10 | 8 | 7 | 0 |
| 4776 | 82 | (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark | Raw | 18 | 14 | 12 | 10 | 8 | 7 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4777 | 01 | NC – MEI Numerical computation (A2) | Raw | 72 | 55 | 47 | 39 | 32 | 25 | 0 |
| | | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4798 | 01 | FPT - Further pure mathematics with technology (A2) | Raw | 72 | 57 | 49 | 41 | 33 | 26 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |

**GCE Statistics (MEI)**

| | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|
| G241 | 01 Statistics 1 MEI (Z1) | Raw | 72 | 59 | 52 | 46 | 40 | 34 | 0 |
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| G242 | 01 Statistics 2 MEI (Z2) | Raw | 72 | 55 | 48 | 41 | 34 | 27 | 0 |
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| G243 | 01 Statistics 3 MEI (Z3) | Raw | 72 | 56 | 48 | 41 | 34 | 27 | 0 |
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |

**GCE Quantitative Methods (MEI)**

| | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|
| G244 | 01 Introduction to Quantitative Methods MEI | Raw | 72 | 58 | 50 | 43 | 36 | 28 | 0 |
| G244 | 02 Introduction to Quantitative Methods MEI | Raw | 18 | 14 | 12 | 10 | 8 | 7 | 0 |
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| G245 | 01 Statistics 1 MEI | Raw | 72 | 59 | 52 | 46 | 40 | 34 | 0 |
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| G246 | 01 Decision 1 MEI | Raw | 72 | 48 | 43 | 38 | 34 | 30 | 0 |
| | | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |

## Level 3 Certificate and FSMQ raw mark grade boundaries June 2016 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

**Level 3 Certificate Mathematics for Engineering**

| | | | Max Mark | a* | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|---|
| H860 | 01 Mathematics for Engineering | | This unit has no entries in June 2016 | | | | | | | |
| H860 | 02 Mathematics for Engineering | | | | | | | | | |

**Level 3 Certificate Mathematical Techniques and Applications for Engineers**

| | | | Max Mark | a* | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|---|
| H865 | 01 Component 1 | Raw | 60 | 48 | 42 | 36 | 30 | 24 | 18 | 0 |

**Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)**

| | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|
| H866 | 01 Introduction to quantitative reasoning | Raw | 72 | 55 | 47 | 39 | 31 | 23 | 0 |
| H866 | 02 Critical maths | Raw | 60 | 47 | 41 | 35 | 29 | 23 | 0 |
| | | Overall | 132 | 111 | 96 | 81 | 66 | 51 | 0 |

**Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)**

| | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|
| H867 | 01 Introduction to quantitative reasoning | Raw | 72 | 55 | 47 | 39 | 31 | 23 | 0 |
| H867 | 02 Statistical problem solving | Raw | 60 | 40 | 34 | 28 | 23 | 18 | 0 |
| | | Overall | 132 | 103 | 88 | 73 | 59 | 45 | 0 |

**Advanced Free Standing Mathematics Qualification (FSMQ)**

| | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|
| 6993 | 01 Additional Mathematics | Raw | 100 | 59 | 51 | 44 | 37 | 30 | 0 |

**Intermediate Free Standing Mathematics Qualification (FSMQ)**

| | | | Max Mark | a | b | c | d | e | u |
|---|---|---|---|---|---|---|---|---|---|
| 6989 | 01 Foundations of Advanced Mathematics (MEI) | Raw | 40 | 35 | 30 | 25 | 20 | 16 | 0 |

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| Version | Details of change |
| --- | --- |
| 1.1 | Correction to Overall grade boundaries for H866 |
| | Correction to Overall grade boundaries for H867 |